ME322 Mechanics of Solids II

SESSION 1: Theories of Stress and Strain

Dr.-Ing. Thira Jearsiripongkul
FACULTY OF ENGINEERING, THAMMASAT UNIVERSITY

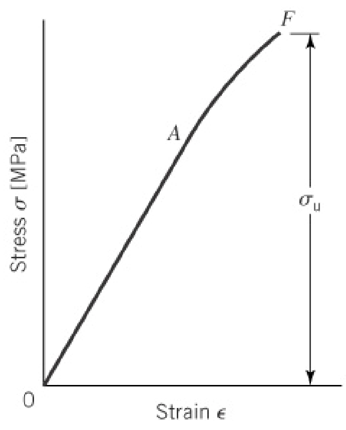

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Fig. 1-13 W-13

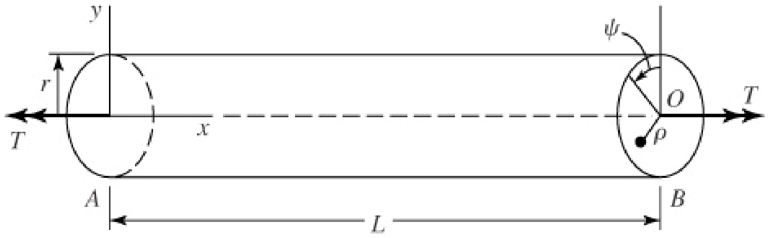

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Fig. 1-3 W-3

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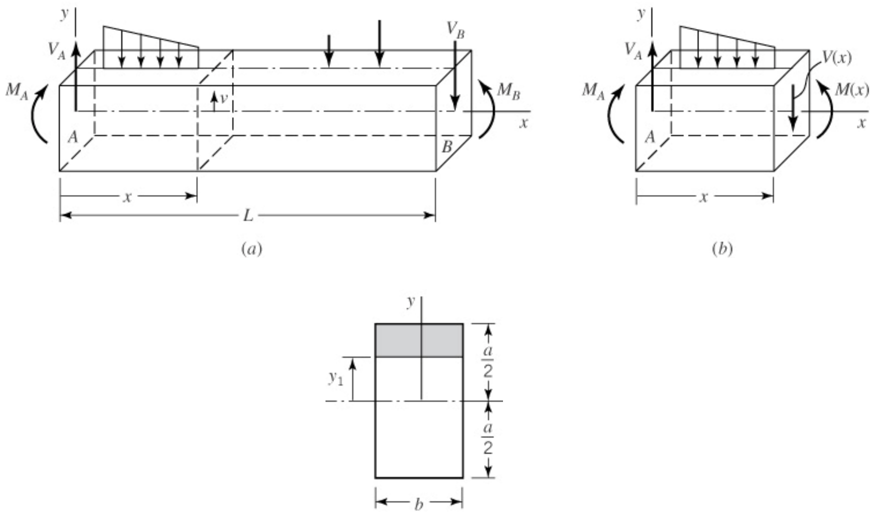

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Definition of stress at a point

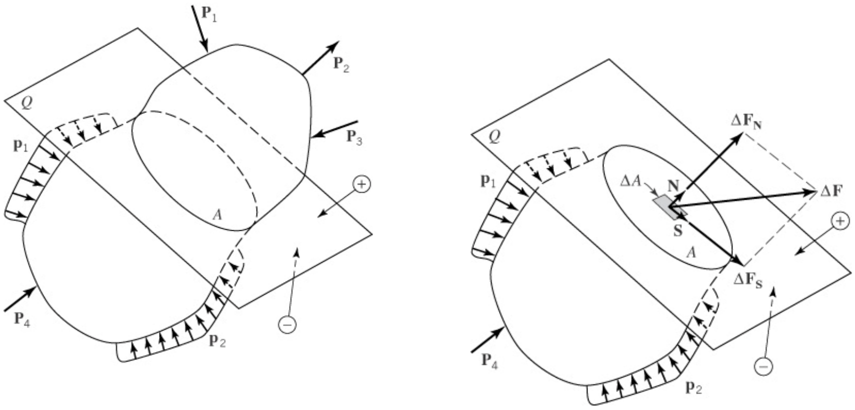

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Fig. 2-1 W-20

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Stress notation

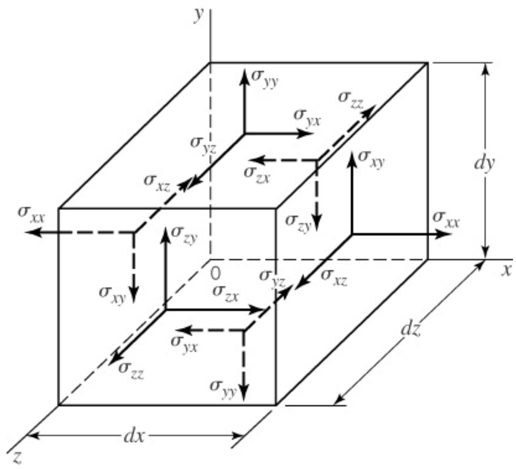

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Fig. 2-3 W-22

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Stresses Acting on Arbitrary Planes

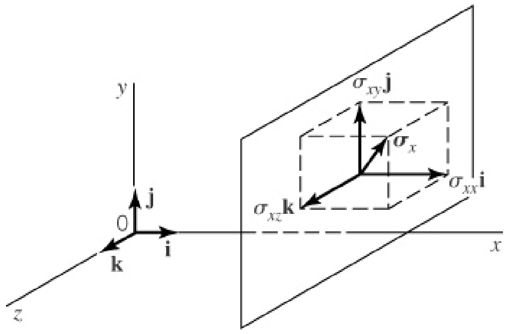

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Fig. 2-5 W-24

Stresses Acting on Arbitrary Planes

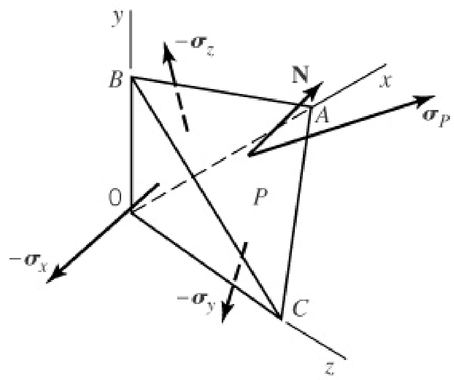

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Fig. 2-6 W-25

Transformation of Stress

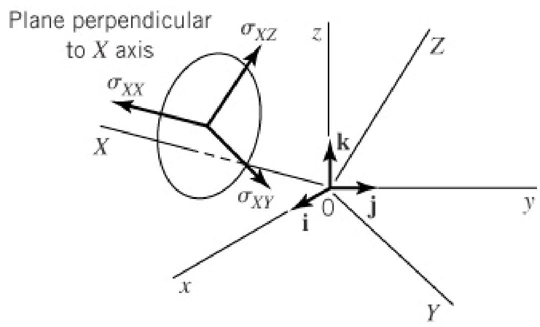

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Fig. 2-8 W-27

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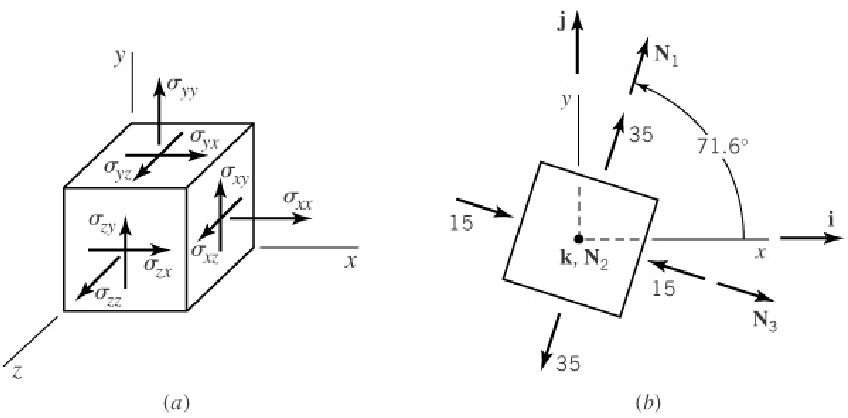
Example: 2.1



The state of stress at a point in a machine part is given by σ_{xx} = -10, σ_{yy} = 30, σ_{xy} = 15 and σ_{zz} = σ_{xz} = σ_{yz} = 0, determine the principal stresses and orientation of the principal axes.

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Example: 2.2

The known stress components at a point in a body, relative to the (x, y, z) axes, are σ_{xx} = 20 MPa, σ_{yy} = 10 MPa, σ_{xy} = 30 MPa, σ_{xz} = -10 MPa, and σ_{yz} = 80 MPa. Also, the second stress invariant is $I_2 = -7800 \text{ (MPa)}^2$

(a) Determine the stress component σ_{zz}. Then determine the stress invariants I_1 and I_3 and the three principal stresses

(b) Show that I_1, I_2, and I_3 are the same relative to (x, y, z) axes and relative to the principal axes (1, 2, 3)

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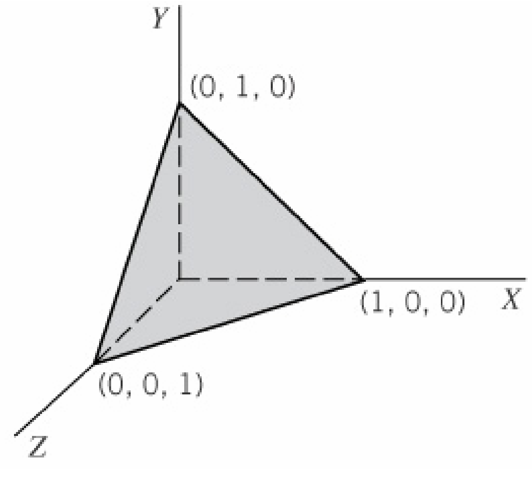
Octahedral Stress



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Location of Transformed Axes for Plane Stress

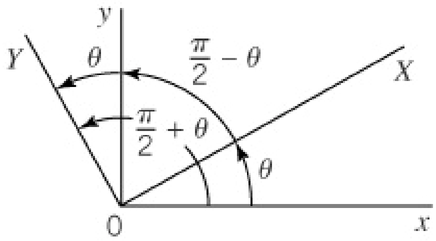

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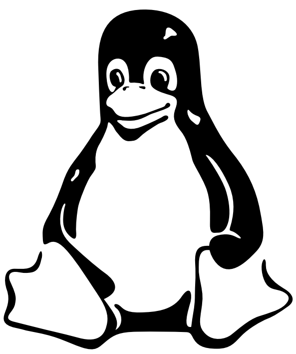
Fig. 2-10 W-30

Problems:

- 2.3
- 2.4
- 2.5
- 2.10
- 2.12

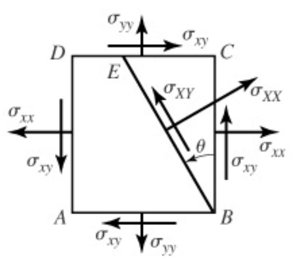


Submission: 23/06/2006
(before 16:00)

Assessment

- Assignments 15% (2 persons for a group)
- Attendance 10% (later than 20 mins: -1%)
- Oral examination 10% (before midterm)
- Midterm 25%
- Oral examination 10% (before final)
- Final 30%

Mohr's Circle (Two Dimensions)



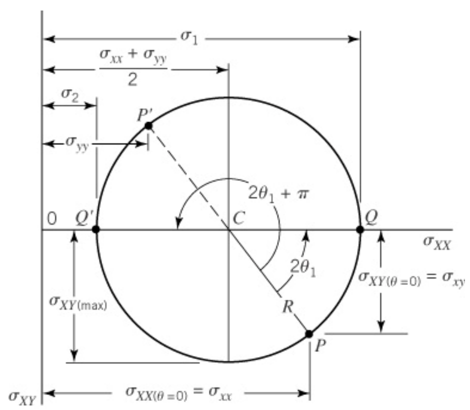


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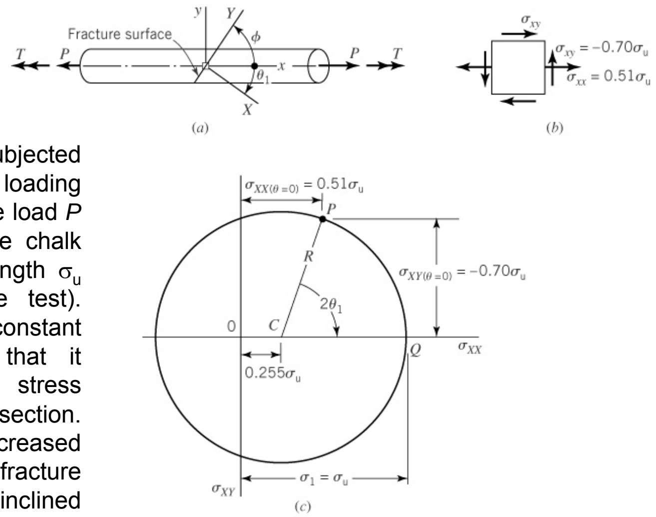
Fig. 2-11 W-31

Example: 2.3



A piece of chalk is subjected to a combined loading consisting of a tensile load *P* and a torque T. The chalk has an ultimate strength σ_u (from simple tensile test). The load *P* remains constant at such a value that it produces a tensile stress $0.51\sigma_u$ on any cross section. The torque is increased gradually until fracture occurs on some inclined surface.

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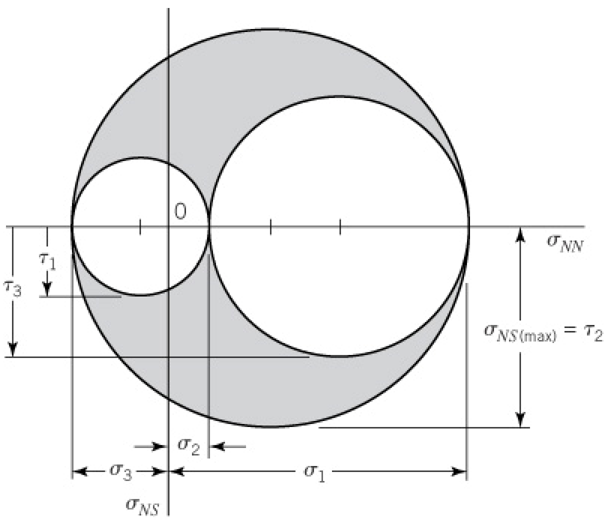
Fig. E2-3 W-33

Mohr's Circle (Three Dimensions)



Example: 2.4

The state of stress at a point in a machine component is given by σ_{xx} = 120 MPa, σ_{yy} = 55 MPa, σ_{zz} = -85 MPa, σ_{xy} = -55 MPa, σ_{xz} = -75 MPa and σ_{yz} = 33 MPa. Construct the Mohr's circles of stress for this state and locate the coordinates of points A and B for normal and shear stress acting on the cutting planes with outward normal vectors given by (relative to the principal axes of stress)

$$N_1 = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right), \quad N_2 = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$$

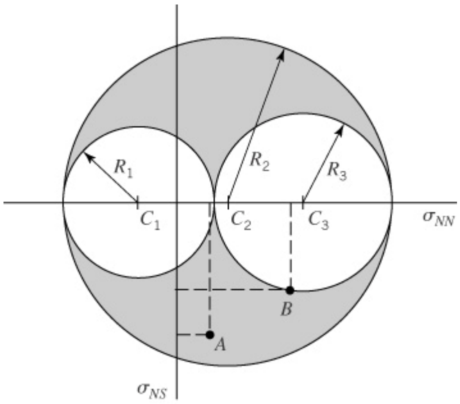


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Fig. E2-4 W-38

• Example 2.5: At a certain point in a drive shaft coupling, the stress components relative to axes (x, y, z) are σ_{xx} = 80 MPa, σ_{yy} = 60 MPa, σ_{zz} = 20 MPa, σ_{xy} = 20 MPa, σ_{xz} = 40 MPa and σ_{yz} = 10 MPa.

(a) Determine the stress vector on a plane normal to the vector **R** = **I** + 2**j** + **k**

(b) Determine the principal stresses σ_1, σ_2, σ_3 and the maximum shear stress τ_{max}

(c) Determine the octahedral shear stress τ_{oct} and compare it to the maximum shear stress

MATLAB:

Command: roots(c)

where c is in vector form

c(1)xn + c(2)xn-1 + … + c(n+1)

Differential Equations of Motion of a Deformable Body

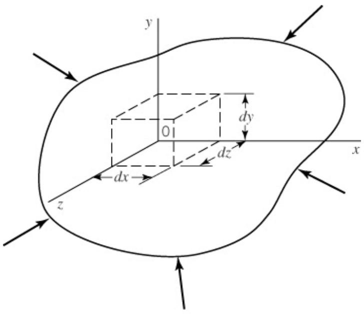

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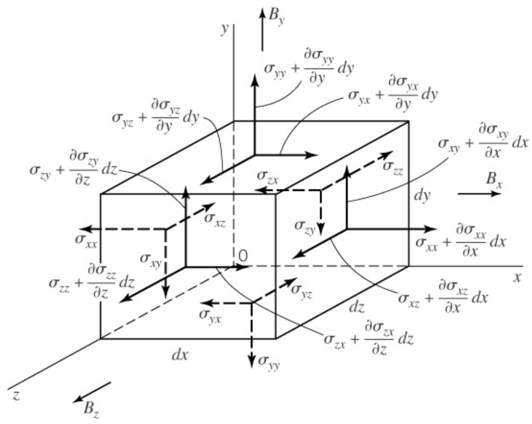
Fig. 2-14 W-34a



Strain Theory: Strain of a Line Element

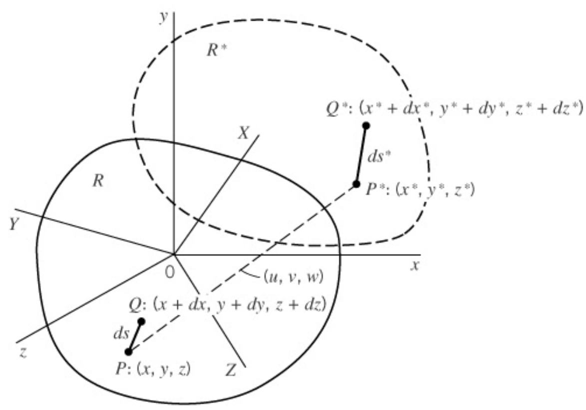


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Fig. 2-18 W-36

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Rotation between Two Line Elements (Definition of Shear Strain)

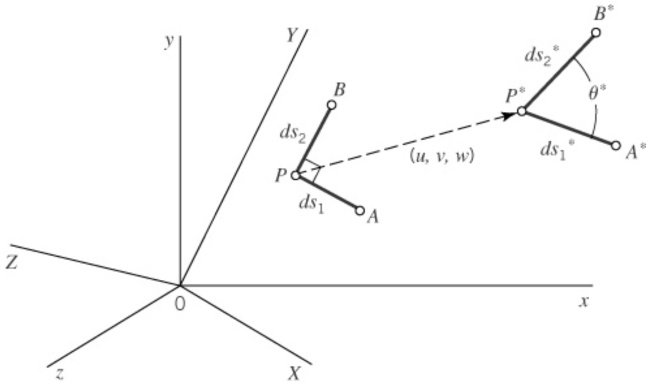

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Fig. 2-19 W-37

- Example 2.8: A machine part is deformed in the shape indicated by the dashed straight lines (small displacement). The displacements are given by the following relations: $u = C_1xyz$, $v = C_2xyz$ and $w = C_3xyz$.

  (a) Determine the state of strain at point E^* for the deformed body are (1.504,1.002,1.996)

  (b) Normal strain at *E* in the direction of line *EA*

  (c) Shear strain at *E* for the undeformed orthogonal line *EA, EF*

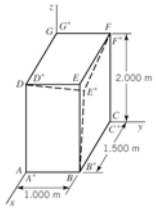

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Fig. E2-8 W-41

• Example 2.9: A straight torsion-tension member with a solid circular cross section has a length L = 6m and radius R = 10mm. The member is subjected to tension and torsion loads that produces an elongation ΔL = 10mm and a rotation of one end of the member with respect to the other end of $\pi/3$ rad. Let the origin of the (r, θ, z) cylindrical coordinate axes lie at the centroid of one end of the member, with the z axis extending along the centroidal axis of the member. The deformations of the member are assumed to occur under conditions of constant volume. The end z = 0 is constrained so that only radial displacements are possible there.

(a) Determine the displacements of any point in the member and the state of strain for a point on the outer surface.

(b) Determine the principal strains for the point where the state of strain was determined.

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• Example 2.10: A state of plane strain ($\varepsilon_{zz} = \varepsilon_{xz} = \varepsilon_{yz} = 0$) at a point in a body is given, with respect to the (x, y, z) axes, as ε_{xx} = 0.00044, ε_{yy} = 0.00016 and ε_{xy} = -0.00008. Determine the principal strains in the (*x*, *y*) plane, the orientation of the principal axes of strain, the maximum shear strain and the strain state on a block rotated by an angle of θ' = 25° measured counterclockwise with respect to the reference axes.

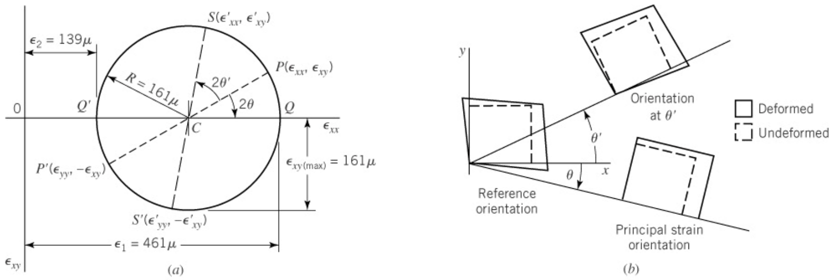

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Fig. E2-10 W-42

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Small-Displacement Theory: Example 2.11

For small-displacement plane strain, the strain components in the plate *ABCD*, in terms of the coordinate system (x, y), are

$$\varepsilon_{xx} = Cy(L-x)$$

$$\varepsilon_{yy} = Dy(L-x)$$

$$\gamma_{xy} = -(C+D)(A^2-y^2)$$

where *A*, *C* and *D* are known constants. The displacement components (*u*, *v*) at *x* = *y* = *0* are zero and the slope at *x* = *y* = *0* is also zero. Determine *u* and *v* as functions of (*x*, *y*)

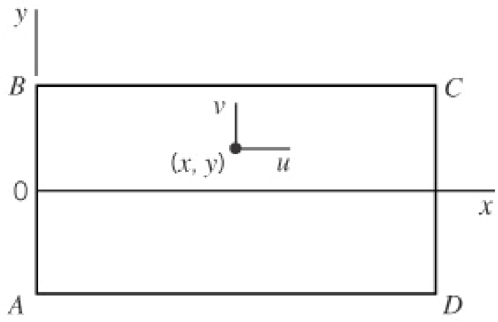

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Fig. E2-11 W-43